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Rent-seeking incentives in share contests☆

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ABSTRACT

This article investigates share contests. In our framework, we allow contestants to have more general preferences than have been used in the literature. Previous approaches have the unfortunate characteristic that contestants' marginal rates of substitution between the rent share allocated by the contest and their effort is constant regardless of the size of the rent share. This results in a conventional wisdom: larger rents command more effort. By providing a more general framework, we show the reverse may also be true and we derive the conditions under which this is the case. Our approach then allows us to rationalize, within a standard contest framework, observations that rents might be more hotly contested when they become scarcer, as has evidently been the case with the recent global contraction of public funds available for public policy.

1. Introduction

Sayre's law: *"In any dispute, the intensity of feeling is inversely proportional to the value of the stakes at issue. That is why academic politics are so bitter."* (Coleman, 2008)

Contests characterize situations in which individuals seek to appropriate an economic rent. This describes a wealth of economic scenarios—such as rent seeking, litigation, and conflict—where the study of contests has improved our understanding of many fundamental economic interactions. The conventional wisdom borne from the analysis of contests suggests that rent-seeking effort is increasing in the size of the rent. Although this is consistent with many applications, there are, however, many other environments in which we might observe that the reverse is true; *Sayre's law*—quoted at the beginning of this introduction—being a case in point.

We focus on rent-seeking incentives in share contests, and motivate our analysis with an application to contests over public funds. In such contests, the contestants are lobbyists who invest effort to obtain a share of a rent that is public funds to provide a public good and we relate the size of this rent, as measured by the amount of available public funds, to rent-seeking efforts. Epstein and Nitzan (2007) argue at length why contests are an appropriate tool for studying public policy and public good provision, while equally providing a host of potential applications. For example, groups may rent seek for investments in health as favored by the elderly (and backed by the pharmaceutical lobbies) as opposed to the young (supported by teachers' associations) who aim at improving education (Cattaneo and Wolter, 2009). The conventional wisdom may apply in such contexts since lobbying groups are typically thought to intensify their efforts in the presence of higher stakes. Yet, as the recent anti-austerity protests and strikes across Europe testify, rent seeking for special interests may very well become more intense in the presence of cuts in government funds. As Reuters (2010) report, "As ministers and civil servants pore over budget books and decide what goes and what stays, an army of lobbyists, consultants, companies and campaigners is fighting to hold the line…". The evidence

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goes beyond anecdotal narratives: a recent study by Ponticelli and Voth (2017) identifies a causal effect of expenditure cuts on social unrest in Europe over the period 1919–2008. The goal of this article, therefore, is to provide a rational explanation for these seemingly contradictory observations that contest effort may either increase or decrease in the size of the rent.

To that end, we develop a novel and general contest theory in which a perfectly divisible rent (e.g., public funds) is shared among contestants (e.g., lobbyists) that have general preferences. Contests in the spirit of Tullock (1980) can be interpreted in two ways: 'winner-take-all' or 'probabilistic' contests; and 'share' contests. In the 'winner-take-all' interpretation there exists a *probability* that a player receives the entire rent based on their relative effort. In a 'share' contest, in contrast, each individual receives a (deterministic) share of the rent based on their effort relative to that of their rivals. Share contests capture lots of important economic scenarios: we focus on contests for public funds to motivate and illustrate ideas in this paper, but there are numerous other applications, for instance, to land conflict and rent seeking over foreign aid (e.g., Svensson, 2000; Skaperdas and Syropoulos, 2002; Hodler, 2007). Despite their wide applicability, share contests have seen relatively little attention in the literature, which has tended to focus on winner-take-all contests with the occasional extension of ideas to share contests. But the two interpretations are fundamentally different in all but the simplest settings. The aim of this paper is to go beyond the simplest setting and explore how share contests work when contestants have, what we will argue are, realistic preferences.

We will denote by R the rent that is being contested; by x^i the effort of contestant i; and by z^i the contest allocation that contestant i receives. Let ϕ for the moment denote the 'contest success function' that depends on the efforts of all contestants. In a winner-take-all contest, ϕ determines the probability of winning the entire rent, whereas in a share contest ϕ determines each contestant's share of the rent. Consider that contestants derive utility from the outcome of the contest and the effort they exert in contesting the rent, captured by $u^i(z^i,x^i)$. Then the appropriate payoff function in a winner-take-all contest is the expected utility $\phi u^i(R,x^i) + [1-\phi]u^i(0,x^i)$, whereas in a share contest the appropriate payoff function is $u^i(\phi R,x^i)$. If u^i is linear so $u^i(z^i,x^i) = z^i - x^i$, or quasi-linear of the form $u^i(z^i,x^i) = z^i - c^i(x^i)$, then these payoffs are the same so share contests and winner-take-all contests are strategically equivalent (Cason et al., 2013), otherwise they command separate study.

Although major advances have been made in developing the analysis of winner-take-all contests to capture non-linear evaluation of contest outcomes by allowing for risk aversion since the contribution of Hillman and Katz (1984)[1], the same is not true of share contests: the two are not equivalent under this extension. Where share contests have been studied in the literature the payoff functions used have either been of the linear or quasi-linear form so in fact the analysis of winner-take-all contests can be transferred to share contests, or where contestants evaluate the net rent so $u^i(z^i,x^i) = v^i(z^i - x^i)$ (see, for instance, Skaperdas and Gan (1995) and Konrad and Schlesinger (1997) whose focus is on winner-take-all contests but include an extension to share contests) where the analysis aligns with the linear case since this is a monotonic transformation of a linear payoff function. These payoff functions share the unfortunate characteristic that the marginal rate of substitution between effort and the contest allocation is the same regardless of the size of the contest allocation. Put another way, the amount of contest allocation a contestant is willing to give up to save a unit of effort is the same no matter how large or how small their allocation from the contest is. We find this restrictive and indeed unrealistic. In our example of contests for public funds, it is highly likely that lobby groups will be much less willing to give up funds to save lobbying effort when funds are scarce than when they are in abundance, so in this application, and indeed in general, we require a theory of contests that allows this marginal rate of substitution to potentially increase in the size of the allocation.

We achieve this by retaining generality in contestants' payoff functions, allowing them to take the form $u^i(z^i,x^i)$ where the marginal rate of substitution between effort and the contest allocation, $MRS^i = -u^i_x/u^i_z$, is not restricted to be constant in z^i, as is the case in the existing literature. We focus initially on simple Tullock contests, and follow the approach of Cornes and Hartley (2003, 2005, 2012) by recognizing and exploiting the aggregative properties of the game that is played. Cornes and Hartley (2005) address the issue of existence and uniqueness of equilibrium in contests with heterogeneous players assuming linear evaluation of the contest allocation; we extend this result to the case of more general preferences, providing sufficient conditions for the existence and uniqueness of Nash equilibrium. We then study the comparative static properties of equilibrium, particularly considering the effect of a change in the size of the contested rent.[2] What is interesting is that when we capture these more realistic preferences the conventional wisdom of a positive relationship between the size of the contested rent and equilibrium rent-seeking effort need no longer hold, but in fact can be reversed: so when rents become scarcer they might be more hotly contested, or when they become more abundant effort goes down. For individual choices, this occurs if the marginal rate of substitution increases sufficiently as z^i increases, as measured by the z^i-elasticity of the marginal rate of substitution that needs to exceed one for effort to decrease in the size of the rent, and we provide a related condition that gives the conditions under which equilibrium aggregate effort declines in the contested rent. This can be true for very standard preferences and requires that contestants have either sufficiently strong diminishing marginal utility over the contest allocation (u^i_{zz} is sufficiently negative), or sufficiently strong substitutability between effort and the contest allocation (u^i_{zx} is sufficiently negative), or a combination of these.

Our analysis of contests with more general preferences means that standard contest theory can now be used to rationalize situations in which increases in contested rents command less effort, or indeed reductions in contested rents command more effort. In the context of our application to contests for public funds, our model makes a substantial contribution to the related literature. While scholars have already focused on rent seeking over public policy and public funds, the majority of studies use a conventional quasi-linear utility setup and focus on questions of heterogeneity in group size and composition (Riaz et al., 1995; Katz and Tokatlidu, 1996; Cheikbossian, 2008), or on comparing rent seeking with the

[1] See, for instance, Long and Vousden (1987), Skaperdas and Gan (1995), Riaz et al. (1995), Konrad and Schlesinger (1997), Treich (2010), Cornes and Hartley (2012), Jindapon and Whaley (2015), Schroyen and Treich (2016), Jindapon and Yang (2017), and Konrad (2009) and Congleton and Hillman (2015) for reviews. Long and Vousden (1987) consider a model in which individuals each contest a rent that they will ultimately receive a share of, but the share is determined randomly, the process being influenced by all contestants' choices of efforts. However, this is not a contest as axiomatized by Skaperdas (1996) since there is nothing to tie the shares of contestants together that would ensure the full rent, and only the full rent, is allocated.

[2] When seeking to understand the comparative static properties of this game, a natural place to turn is the literature on aggregative games. Corchón (1994) investigates the comparative static properties of aggregative games in a general setting but assumes the game is one of strategic substitutes. These results do not apply to contests as they are neither games of strategic substitutes nor strategic complements. Acemoglu and Jensen (2013) consider a more general setting and provide sufficient conditions for comparative statics to be—following their terminology—'regular' in 'nice' aggregative games by considering particular changes in the game termed 'positive shocks'. However, while all of the conditions are satisfied for contests with a linear evaluation of the contest outcome, their 'positive shocks' framework is not suited to the study of contests with heterogeneous contestants that have more general preferences, rendering a bespoke analysis of this framework necessary.

alternative of a market (Gradstein, 1993). By imposing such specific functional forms, however, the literature has implicitly constrained the marginal rate of substitution between public funds share and contest (or lobbying) effort: the marginal rate of substitution is assumed to be insensitive to the amount of public funds allocated. Yet, it seems reasonable to consider cases where marginal rates of substitution may change. For example, professional bodies may deploy a milder effort to protect their rights when they already enjoy generous conditions (in terms of retirement age, tax reliefs, in-kind benefits, and so on) than when they do not. Using our approach highlights the role of preferences on equilibrium levels of rent seeking and provides a theoretical justification for intensive rent seeking over scarce public funds, as exemplified by the increasing amount of strikes and protests in periods of budget cuts.[3]

The remainder of the article is structured as follows. In Section 2, we outline share contests in which contestants have general preferences, and we go on to analyze the existence and uniqueness of Nash equilibrium by exploiting the aggregative properties of the game in Section 3. In Section 4, we explore the relationship between the size of the contested rent and contestants' effort in equilibrium. Section 5 provides numerous further applications of our model and Section 6 provides our concluding remarks. All proofs are contained in the Appendix and a supplementary online Appendix considers the dissipation ratio, and extensions of the model to more general contest success functions and endogenous determination of the rent.

2. The model and intuition

Consider a set of individual contestants (e.g., lobbyists, politicians) $N = \{1, \ldots, n\}$, $n \geq 2$, which participate in a share contest to obtain an economic rent R (e.g., public funds). Contestant i's share of the rent is determined by their effort relative to the effort of other contestants and is given by the contest success function $\phi(x^i, \mathbf{x}^{-i})$, where $x^i \geq 0$ denotes the effort of contestant $i \in N$ and $\mathbf{x}^{-i}$ denotes the vector of all other contestants' effort levels. Define z^i as being contestant i's allocation of the rent from the contest:

$$z^i \equiv \phi(x^i, \mathbf{x}^{-i})R. \tag{1}$$

We begin by studying a 'simple' Tullock contest for an exogenously-given rent of size R in which

$$\phi(x^i, \mathbf{x}^{-i}) = \begin{cases} \frac{x^i}{x^i + X^{-i}} & \text{if } X > 0 \text{ or} \\ \frac{1}{n} & \text{otherwise,} \end{cases} \tag{2}$$

where $X \equiv \sum_{j \in N} x^j$ is the aggregate effort of all contestants and $X^{-i} \equiv X - x^i$.[4]

For each contestant i we define a utility function $u^i(z^i, x^i)$ over their contest allocation, z^i, and their effort in contesting the rent, x^i. We denote by $MRS^i(z^i, x^i)$ contestant i's marginal rate of substitution between z^i and x^i:

$$MRS^i(z^i, x^i) \equiv -\frac{u^i_x}{u^i_z},$$

which gives the amount of additional allocation from the contest that is required to compensate for an incremental increase in effort.[5]

As previously mentioned, the literature has so far only considered either (quasi-)linear preferences in such contests or contestants evaluating the contest allocation net of effort. Study of these models has provided the conventional wisdom of a monotonically increasing relationship between the contested rent and contest effort. But these limited payoff functions have an important and we think unrealistic property: the marginal rate of substitution between the contest allocation and effort is constant regardless of the size of the contest allocation. In many applications, such as with rent seeking over public funds, it may appear unrealistic to assume a constant marginal rate of substitution: if this is the case then whether you have one unit of contest allocation, or 1000 units, the amount you are willing to give up for one less unit of effort is identical. Yet if public funds are scarce, you may not be willing to give up much at all to save lobbying effort, but when they are plentiful you are less concerned: thus, the marginal rate of substitution is not constant, but is increasing in the size of the contest allocation. It is important to recognize this because if the marginal rate of substitution is allowed to increase in the size of the contest allocation, as we will see equilibrium contest efforts may decrease in the size of the contested rent. This then turns the conventional wisdom on its head and allows us, within a standard contest framework just with more realistic preferences, to rationalize rents being more hotly contested when they become scarcer, and indeed contestants being less effortful when rents increase.

To further explore the intuition behind this finding, note that in a Nash equilibrium in a simple Tullock contest each contestant may be seen as solving the problem

$$\max_{z^i, x^i \geq 0} u^i(z^i, x^i) \text{ s.t. } z^i = \frac{x^i}{x^i + X^{-i}} R.$$

For a fixed X^{-i}, this optimization problem can be represented graphically in the (x^i, z^i)-space by considering the point on the budget constraint—which is an increasing and concave function that starts from the origin—that puts the individual on the most north-westerly indifference curve derived from the utility function. Thus, we seek a level of effort—denoted by $\tilde{x}^i$—where the marginal rate of substitution of contest allocation for effort is equal to the slope of the budget constraint, which is $\frac{X^{-i}}{[x^i + X^{-i}]^2} R$. With a linear payoff function $u^i(z^i, x^i) = z^i - x^i$ and the marginal rate of substitution is everywhere equal to 1; and with linear evaluation of the rent but a convex cost of effort, i.e., $u^i(z^i, x^i) = z^i - c^i(x^i)$, the marginal rate of substitution is $c^{i'}(x^i)$. In each of these cases, indifference curves are vertical displacements of each other as their slope does not depend on the contest allocation.

Define a *rent expansion path* as the points that trace out the optimal effort-contest allocation combination for a contestant when the rent increases (keeping fixed the actions of all other contestants). This is illustrated in Fig. 1 by aa' when the rent increases from R to R' for a variety of preferences. From Fig. 1, we can observe that with a higher rent the slope of the budget constraint increases everywhere. With linear preferences (illustrated in Panel (a)) indifference curves are straight parallel lines and so in this case the former optimal

[3] Epstein and Nitzan (2006) use a two-player contest to investigate how rent-seeking effort changes when a public policy is accepted/rejected. They show that a more 'restrained' government intervention (rent reduction) may increase efforts due to the asymmetry of stakes between both players: the characteristics of the marginal stakes are shown to affect the net benefit that both contestants face. It is noteworthy that one cannot obtain *both* players increasing their contest effort for a rent reduction; their result is driven by the increased asymmetry in prize valuation that attenuates equilibrium efforts. In our general framework we demonstrate that all contestants may have incentives to increase their contest efforts for lower prizes.

[4] In a supplementary online Appendix, we consider more general contest success functions, as well as situations in which the size of the rent is endogenously determined by contestants' effort and derive similar results.

[5] In a conventional contest model with quasi-linear preferences, $u^i(z^i, x^i) = z^i - c^i(x^i)$ and so $MRS^i(z^i, x^i) = c^{i'}(x^i)$.

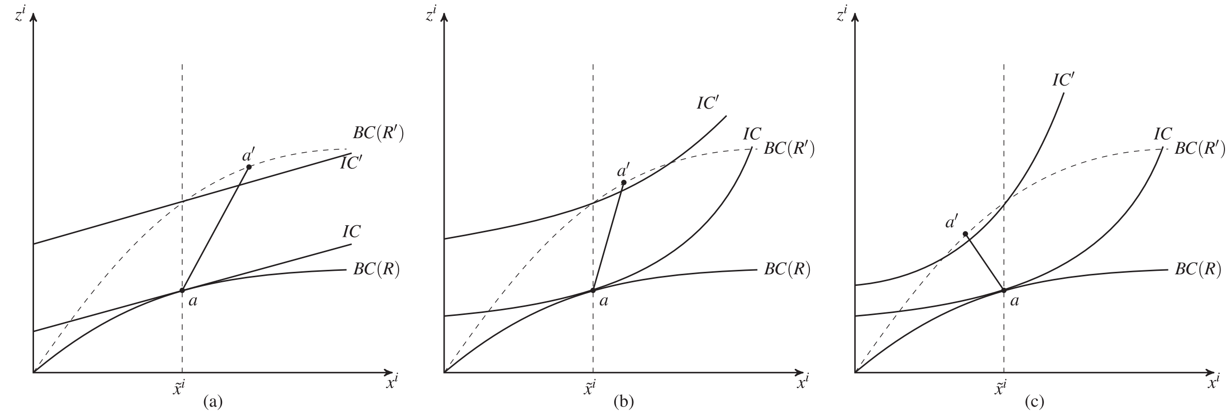

Fig. 1. Rent expansion paths aa' with different preferences. In Panel (a) the slope of indifference curves do not depend on z^i (as would also be the case for quasi-linear preferences), so the rent expansion path is upward-sloping; in Panel (b) the marginal rate of substitution increases, but not by as much as the slope of the budget constraint, so the rent expansion path is again positively-sloped; in Panel (c) the marginal rate of substitution increases by more than the slope of the budget constraint, giving a negatively-sloped rent expansion path.

effort can no longer be optimal on the new budget constraint since at this allocation the slope of the indifference curve must be less than the slope of the (now steeper) budget constraint. This necessitates an increase in effort to regain tangency, hence tracing out a positively-sloped rent expansion path. The same is true with quasi-linear preferences in which the cost of effort is convex and there is linear evaluation of the contest allocation. In this case (not illustrated), the slope of the indifference curve does not depend on z^i: at the former optimal level of x^i on the new budget constraint the slope of the indifference curve must be less than that of the budget constraint, again necessitating an increase in optimal effort.

If, on the other hand, the marginal utility of the contest allocation is sufficiently decreasing in z^i, or there is sufficiently strong substitutability between effort and the contest allocation, or both, then when z^i increases with x^i fixed, the marginal rate of substitution increases. If, as in Panel (b) of Fig. 1, the marginal rate of substitution increases by less than the increase in the slope of the budget constraint then, again, the rent expansion path will be positively-sloped, consistent with the conventional wisdom on contests. Conversely, if the marginal rate of substitution increases by more than the increase in the slope of the budget constraint, then with the higher rent optimality will occur at a *lower* level of effort, giving a negatively-sloped rent expansion path, as shown in Panel (c) of Fig. 1.[6]

This discussion highlights that taking a non-constant marginal rate of substitution between contested rent and contest effort into account would seem to be very important. But of course, so far this is just an analysis of best responses. To understand when the conventional wisdom in contests holds, and when it does not, we must develop an understanding of the general conditions on preferences under which individual and aggregate effort *in equilibrium* increases or decreases with the contested rent, which is a key aim of this article and what we turn to in the next section.

To complete the model setup, we impose the following assumption.

[6] This can also be observed by using, for example, a CES function over leisure and controlled rent-share where contestants have a homogeneous time endowment e that they can either enjoy directly (leisure), or else devote to appropriation activities. It is straightforward to show for the symmetric equilibrium that when leisure and z^i are perfect complements (substitutes) then equilibrium efforts are decreasing (increasing) in the rent. Further, for a Cobb-Douglas set up, equilibrium effort is independent of the rent.

Assumption 1. For each $i \in N$,

(a) the utility function is twice continuously differentiable with $u_z^i > 0$, $u_x^i \le 0$ with a strict inequality if $x^i > 0$, and $u_{zx}^i \le \min\left\{-MRS^i u_{zz}^i, -\frac{1}{MRS^i}u_{xx}^i\right\}$ (i.e., $u_{zz}^i \le -\frac{1}{MRS^i}u_{zx}^i$ and $u_{xx}^i \le -MRS^i u_{zx}^i$); and

(b) $MRS^i(z^i, 0) < \infty$ for all $z^i \ge 0$, and if $MRS^i(0,0) = 0$ then $MRS_x^i(0,0) > 0$.

Further, for any R there is a contestant $i \in N$ for whom there exists an $\epsilon > 0$ such that $u^i(R, \epsilon) - u^i(R/n, 0) > 0$, a sufficient condition for which is $u_x^i(z^i, 0) = 0$ for all $z^i \ge 0$.

We assume contestants are never satiated with respect to the contest allocation, and effort is always distasteful. Condition (b) rules out contestants always being inactive in any contest and always wanting to exert infinite effort. The final condition ensures there is no null equilibrium in which all contestants exert zero effort, since there is at least one contestant that would prefer to acquire the whole rent for a small effort than be awarded an equal share of the rent for no effort. The curvature restrictions we impose in part (a) give rise to various properties that we collect together in the following lemma.

Lemma 1. *Under the conditions of Assumption 1 part (a),*

1. $MRS_z^i \ge 0$ *and* $MRS_x^i \ge 0$*;*
2. $u^i(\cdot,\cdot)$ *is quasi-concave; and*
3. *If* $u_{zx}^i = 0$, $u^i(\cdot,\cdot)$ *is concave.*

Assumption 1, which we suppose is satisfied in the remainder of the analysis, allows for linear preferences where $u^i = z^i - x^i$, as well as a convex cost of effort if we specified $u^i(z^i, x^i) = z^i - c^i(x^i)$ with $c^{i\prime} > 0, c^{i\prime\prime} \ge 0$.[7] In addition, it permits a very broad class of preferences, allowing us to capture the effects of both diminishing

[7] The standard Tullock contest is thus nested within our setup since such preferences can be imposed by assuming $u_{zx}^i = 0$ (additive preferences) and $u_{zz}^i = 0$ (linear valuation of the contest allocation). Strict convexity of the cost function is accommodated by imposing $u_{xx}^i < 0$.

marginal utility and interactions between the level of effort a contestant uses and their (marginal) valuation of the contest allocation. By considering more general preferences, our framework can not only provide an analysis that nests previous studies of share contests but also provides a tractable methodology by which to consider a broader class of preferences, which can be used to advance and expand the understanding and applicability of contests.

3. Characterizing equilibria in Tullock contests with general preferences

We now turn to characterize equilibria in a simple Tullock contest over an exogenously-given perfectly divisible rent R. We seek a Nash equilibrium in the simultaneous-move game of complete information in which the player set is the contestants $N = \{1, \ldots, n\}$; their strategies are their choice of effort $x^i \geq 0$; and their payoffs are given by their utility of the contest outcome $u^i(z^i, x^i)$ that we assume satisfies Assumption 1, where $z^i = \phi(x^i, \mathbf{x}^{-i})R$ with $\phi(x^i, \mathbf{x}^{-i})$ specified in Eq. (2).

At a Nash equilibrium of the contest, each player may be seen as solving the problem

$$\max_{z^i, x^i \geq 0} u^i(z^i, x^i) \text{ s.t. } z^i = \frac{x^i}{x^i + X^{-i}} R.$$

Since the objective function is strictly monotonic and quasi-concave, the fact that the constraint is quasi-convex (in fact strictly quasi-convex, since $\frac{x^i}{x^i+X^{-i}}R$ is a strictly concave function of x^i) means the first-order condition is both necessary and sufficient in identifying best responses, which are characterized by the tangency condition

$$MRS^i\left(\frac{x^i}{x^i + X^{-i}}R, x^i\right) \geq \frac{X^{-i}}{[x^i + X^{-i}]^2}R, \tag{3}$$

with equality if $x^i > 0$. As such, if $MRS^i(0,0) \geq R/X^{-i}$ the solution is at $x^i = 0$ while if $MRS^i(0,0) < R/X^{-i}$ the solution is given by the above expression with the inequality replaced with an equality.[8] We denote by $b^i(X^{-i}; R)$ the best response function of contestant i.

Rather than working directly with best responses, we turn to analyze the contest using a 'share function' approach that exploits the aggregative nature of the game, extending the result of Cornes and Hartley (2005), that assumes linear evaluation of the contest outcome, to the case of general preferences that satisfy Assumption 1. By way of motivation, this approach differs from pursuing study of best responses in the following way: rather than asking what value of contestant i's effort is consistent with a Nash equilibrium in which the aggregate effort of all other contestants is X^{-i} (which is the best response), it asks what value of individual effort is consistent with a Nash equilibrium in which the aggregate effort of *all* contestants, including contestant i, is X. This gives individual consistency, and to identify a Nash equilibrium aggregate consistency is required, where the sum of individual efforts is exactly equal to the aggregate effort. Rather than working with effort *levels*, it is natural in share contests to work with *shares* of the aggregate effort, in which case the aggregate consistency condition requires the sum of the shares to be equal to 1.

For each contestant, define a 'share function' that gives their share of the rent that is consistent with a Nash equilibrium in which the aggregate effort of all contestants is $X > 0$. By replacing X^{-i} with $X - x^i$ in the first-order condition (3), letting $\sigma^i \equiv x^i/X$ and then replacing x^i with $\sigma^i X$, we deduce that contestant i's share function is given by the value of $\sigma^i \in [0, 1]$ such that $MRS^i(\sigma^i R, \sigma^i X) \geq [1 - \sigma^i][R/X]$, where the inequality is replaced with an equality if $\sigma^i > 0$. As such, accounting for corner solutions, we can write the share function as $s^i(X; R) = \max\{0, \sigma^i\}$ where σ^i is the solution to

$$l^i(\sigma^i, X; R) \equiv MRS^i(\sigma^i R, \sigma^i X) - [1 - \sigma^i]\frac{R}{X} = 0. \tag{4}$$

Share functions shed light on individual behavior consistent with a Nash equilibrium: $Xs^i(X; R)$ is the effort of contestant i consistent with a Nash equilibrium in which the aggregate effort of all contestants is $X > 0$.

The following lemma sets out the properties of individual share functions.

Lemma 2. *For each contestant $i \in N$,*

1. *$s^i(X; R)$ is a continuous function defined for all $X > 0$ and R;*
2. *(a) $s^i(X; R) \to 1$ as $X \to 0$; and (b) either $s^i(X; R) = 0$ for all $X \geq \bar{X}^i(R) \equiv R/MRS^i(0,0)$ if $MRS^i(0,0) > 0$ or, if $MRS^i(0,0) = 0$, $s^i(X; R) \to 0$ as $X \to \infty$; and*
3. *$s^i(X; R)$ is positive for $0 < X < \bar{X}^i$ if $MRS^i(0,0) > 0$ or for all $0 < X < \infty$ if $MRS^i(0,0) = 0$, where it is strictly decreasing in X.*

Note that if $MRS^i(0,0) > 0$ there is some 'drop-out' value of aggregate effort $\bar{X}^i(R)$ where the contestant would become inactive in the contest; whereas if $MRS^i(0,0) = 0$ they would be active in any contest.

As noted, identification of a Nash equilibrium requires aggregate consistency, that is, the sum of individual share functions to be equal to unity. Letting

$$S(X; R) \equiv \sum_{j \in N} s^j(X; R),$$

we have the following equivalence statement.

Lemma 3. *In a contest with rent R, there is a Nash equilibrium with aggregate effort $X^* > 0$ if and only if*

$$S(X^*; R) = 1.$$

Questions of the existence and uniqueness of Nash equilibrium now rest on consideration of the behavior of the aggregate share function $S(X; R)$, whose properties are derived from individual share functions, and its intersection with the unit line. The properties of individual share functions imply that in a contest in which the rent is R the aggregate share function $S(X; R)$, being constructed from a sum of at least two individual share functions, exceeds 1 when X is small enough, is less than one when X is large enough, and is continuous and strictly decreasing in X implying there is exactly one value of $X > 0$ where $S(X; R) = 1$. Since the final statement in Assumption 1 rules out there being an equilibrium with $X = 0$ since

[8] Notice that since $MRS^i(z^i, x^i) = -\frac{u^i_x}{u^i_z}$, $MRS^i(0,0) = 0$ obtains if $u^i_x(0,0) = 0$, i.e., marginal disutility of effort is zero for $x^i = 0$, or $\lim_{z^i \to 0} u^i_z(z^i, 0) = +\infty$, i.e., the utility function satisfies an Inada condition. In a conventional setup with quasi-linear utility, $MRS^i(z^i, x^i) = c^{i\prime}(x^i)$ so $MRS^i(0,0) = 0$ requires the marginal cost of contest effort to be zero for $x^i = 0$.

there is always one contestant that wishes to be active, this allows us to conclude there is a unique Nash equilibrium.

Proposition 1. *In a contest with rent R, there is a unique Nash equilibrium with aggregate effort $X^* > 0$ such that*

$$S(X^*; R) = 1.$$

The set of active contestants is $\mathcal{N}(R) = \{i \in N : \bar{X}^i(R) > X^*\}$, and the equilibrium effort of contestant $i \in \mathcal{N}(R)$ is $x^{i^*} = X^* s^i(X^*; R)$.

Notice that if $MRS^i(0, 0) = 0$ for all contestants then in the Nash equilibrium all contestants will be active, whereas if there are some contestants for whom $MRS^i(0, 0) > 0$ these contestants may be inactive in equilibrium depending on how the equilibrium aggregate effort relates to their 'drop-out' value $\bar{X}^i(R)$.

Proposition 1 confirms that in rent-sharing contests where players can have more general preferences over their allocation of the rent and the effort exerted in contesting the rent (but that nevertheless satisfy Assumption 1), the uniqueness of Nash equilibrium—as found in simple Tullock contests with linear preferences—is preserved.

4. The effect of changing the size of the contested rent

We now turn to investigate how contestants' equilibrium behavior depends on the size of the rent they are contesting.[9] Our method for finding the Nash equilibrium relies on identifying the value of aggregate effort where the aggregate share function equals unity. We write $\mathcal{X}(R)$ for the equilibrium aggregate effort in a contest where the size of the rent is R, which satisfies

$$S(\mathcal{X}(R); R) = 1. \tag{5}$$

Having exploited the aggregative properties of the game, it is relatively straightforward to deduce how a change in the contested rent affects the equilibrium aggregate effort by considering how the aggregate share function, which is a simple sum of individual share functions that is decreasing in aggregate effort, changes at equilibrium when the rent changes.

The relationship between individual share functions and the contested rent depends on the z-elasticity of the marginal rate of substitution, measuring the responsiveness of the marginal rate of substitution to changes in the contest allocation, given by

$$\eta^i \equiv \frac{z^i MRS^i_z}{MRS^i}, \tag{6}$$

and we write $\eta^i(R)$ for the equilibrium elasticity of contestant i in a contest in which the rent is R. The following lemma elucidates the aforementioned relationship.

Lemma 4. *Where contestant i's share function is positive,*

$$s^i_R \gtreqless 0 \Longleftrightarrow \eta^i \lesseqgtr 1.$$

Suppose there is a contest in which all contestants are active, and consider a change in the contested rent. If the marginal rate of substitution is z-inelastic (elastic) for all contestants at equilibrium, i.e., $\eta^i(R) < (>) 1$ for all $i \in N$, then with a larger rent all individual share functions and therefore the aggregate share function will increase (decrease) at $\mathcal{X}(R)$ and consequently the new equilibrium aggregate effort will be higher (lower) than previously. We, however, allow for heterogeneous agents whose response to an increase in the rent may be different. Rather than require all contestants to have an elastic (inelastic) marginal rate of substitution, our necessary and sufficient condition involves an appropriately-weighted sum of contestants' elasticities.

Proposition 2. *Suppose that $MRS^i(0, 0) = 0$ for all $i \in N$. Then*

$$\mathcal{X}'(R) \gtreqless 0 \Longleftrightarrow \sum_{j \in N} w^j [\eta^j(R) - 1] \lesseqgtr 0$$

where $w^i = MRS^i \left[R \left[R\, MRS^i_z + X\, MRS^i_x + \frac{R}{X} \right] \right]^{-1} > 0\,.$

If we relax the assumption $MRS^i(0, 0) = 0$ for all $i \in N$ then some contestants may be inactive in equilibrium. This presents two issues: if $\bar{X}^i(R) = \mathcal{X}(R)$ for any $i \in N$ then the aggregate share function has a kink at the equilibrium; and if we consider a change in the rent that engenders a reduction in aggregate effort this may be reversed by previously inactive contestants becoming active. The following proposition relaxes the assumption that all contestants are active in equilibrium, but at the cost of imposing some structure on the equilibrium.

Proposition 3. *Consider a contest with rent R in which the set of active contestants is $\mathcal{N}(R) = \{i \in N : \bar{X}^i(R) > \mathcal{X}(R)\}$ and assume that $\bar{X}^i \notin [\mathcal{X}(R) + \min\{0, \mathcal{X}'(R)\}, \mathcal{X}(R)]$ for all $i \in N \backslash \mathcal{N}(R)$. Then*

$$\mathcal{X}'(R) \gtreqless 0 \Leftarrow \sum_{j \in \mathcal{N}(R)} w^j [\eta^j(R) - 1] \lesseqgtr 0.$$

With a larger contested rent (e.g., an increase in contestable public funds) whether effort increases, as the conventional wisdom suggests, or decreases depends on whether contestants' marginal rate of substitution (e.g., lobbyists' marginal rate of substitution between the share of public funds and lobbying effort) is z-inelastic or elastic. If a contestant's marginal rate of substitution is z-elastic then when z^i changes the proportional change in the marginal rate of substitution is larger than the proportional change in z^i which implies that the ratio MRS^i/z^i increases, so the marginal rate of substitution not only increases, but increases by an amount sufficient to increase the ratio. In conventional analysis of contests, this does not happen since the marginal rate of substitution is assumed to be constant in z^i, but it can happen with very reasonable preferences. Returning to our diagrammatic exposition, this condition makes perfect

[9] This is neither a game of strategic substitutes nor strategic complements. To see this, define $\tilde{\psi}^i(x^i, X^{-i}) = u^i_z \frac{X^{-i}}{[x^i + X^{-i}]^2} R + u^i_x$ as the marginal payoff of contestant i. Strategic substitutability (complementarity) requires this marginal payoff to be decreasing (increasing) in the aggregation of other contestants' actions. But here $\tilde{\psi}^i_{X^{-i}} = u^i_z \frac{x^i - X^{-i}}{[x^i + X^{-i}]^3} R - \frac{x^i}{[x^i + X^{-i}]^2} R \left[u^i_{zz} \frac{X^{-i}}{[x^i + X^{-i}]^2} R - u^i_{zx} \right]$, the sign of which is clearly ambiguous. Thus, the literature that assumes a particular strategic nature of the game—such as Corchón (1994)—cannot be applied here. Acemoglu and Jensen (2013) show that in 'nice' aggregative games that satisfy their 'local solvability condition' comparative statics are 'regular' in a framework of 'positive shocks'. Writing the marginal payoff of contestant i as a function of the aggregate actions and the contested rent $\psi^i(x^i, X, R) = u^i_z \frac{X - x^i}{X^2} R + u^i_x$, 'positive shocks' would require that $\psi^i_R > 0$ (everywhere). With separable preferences and linear evaluation of the contest allocation (as studied in the literature) $u^i_z = 1$ and $u^i_{zz} = u^i_{xz} = 0$ so $\psi^i_R = \frac{X - x^i}{X^2} > 0$. However, in our framework of more general preferences $\psi^i_R = u^i_z \frac{X - x^i}{X^2} + \frac{x^i}{X} \left[u^i_{zz} + u^i_{xz} \right]$, whose sign is both ambiguous and may differ between players meaning the results of Acemoglu and Jensen (2013) cannot be directly applied to our setting, so we require a bespoke analysis.

sense: when the rent increases the change in the slope of the budget constraint is given by $[1-\sigma^i][1/X]$ which, using the first-order condition, is equal to MRS^i/R; the change in the slope of indifference curves is $\frac{\partial}{\partial R}\{MRS^i(\sigma^i R, \sigma^i X)\} = \sigma^i MRS^i_z$; as such, at the original optimal effort on the new budget constraint, the slope of the indifference curve increases by more (less) than the slope of the budget constraint—giving rise to a reduction (increase) in effort—precisely if $z^i MRS^i_z - MRS^i = \eta^i - 1 > (<) 0$.

In terms of derivatives of the utility function,

$$\eta^i = -\frac{z^i\left[u^i_x u^i_{zz} - u^i_z u^i_{zx}\right]}{u^i_z u^i_x}.$$

Recalling that $u^i_x < 0$, with sufficiently strong diminishing marginal utility of the contest allocation (i.e., u^i_{zz} sufficiently negative), or a sufficiently negative interaction between effort and the marginal utility of the contest outcome (i.e., u^i_{zx} sufficiently negative), or indeed an appropriate combination of both, contestants can exhibit $\eta^i > 1$ which can give rise to the negative relationship between the size of the contested rent and contest effort identified above that contrasts with the conventional wisdom. In the conventional analysis of contests studied in the literature with linear or quasi-linear utility, $u^i_{zz} = u^i_{zx} = 0$ and so $\eta^i = 0$; and where the net rent is evaluated $u^i_x = -u^i_z$ and $u^i_{zx} = -u^i_{zz}$ and so again $\eta^i = 0$: contestants' marginal rate of substitution is therefore never z-elastic.

5. Other applications

We motivated our paper with a contest over public funds that can be modeled as a share contest where the contestants are lobbyists, the rent is the total amount of public funds, contest effort is lobbying effort and the allocation from the contest is the share of public funds the lobbyists receive. The allocation from the contest z^i and contest effort x^i enter individuals' payoff functions as two separate arguments so payoffs are written as $u^i(z^i, x^i)$ with $u^i_z > 0$ and $u^i_x < 0$. The model and results presented are relevant to many analogous environments where rents or resources are divisible; no markets exist for the resource; and contestants care explicitly about the cost of effort.[10]

There are multifarious environments in which these criteria are satisfied, where our model presents a meaningful approach to considering resource allocation. Our results suggest that in these environments, it is very important to consider the nature of contestants' preferences, for if their preferences exhibit an allocation-elastic marginal rate of substitution, changes in the size of the contested resource may have effects that contrast with what was previously understood about contests. Examples that we will discuss in this section include rent seeking over public policy; land conflict; rent seeking over foreign aid; and campaign spending.

5.1. Rent seeking over public policy

Our framework can assist in studying rent seeking over a public policy continuum (Epstein and Nitzan, 2007). For example, consider the question of environmental regulation where NGOs lobby against industrialists (Binder and Neumayer, 2005). While the preferred policy of the contestants are a single and well-defined public policy, the actual policy decision may lie anywhere in between these two policies to the extent that the implemented regulation can, for example, nuance the level of CO_2 emissions. Similarly, in matters of immigration policy where the interests of labor unions and of business interest groups diverge (Facchini et al., 2011), immigration quotas will range over a continuum. Using our model, one could view the distance between preferred policies as the total rent at stake, with each contestant attempting to minimize the distance between the implemented policy and his/her preferred policy like in standard voting models *à la* Persson and Tabellini (2000). The (dis)utility of the implemented policy need not be linear in the distance from one's preferred policy, thus implying that our general framework is well-suited for studying this wide class of games. It seems very natural to consider that lobbyists will have a non-linear evaluation of this outcome, and it would not be unreasonable to consider that they are more sensitive to concessions close to their ideal than incremental concessions when the policy is further from their ideal.[11] Our results suggest that there may not always be a monotonic relationship between lobbying effort and the distance between lobbyists' preferred policies, but that contests between lobbyists with closely aligned preferences may also be bitterly fought.

5.2. Land conflict

Our model is well suited for studying land-related conflicts, and can contribute to an explanation for the ambivalent effect of land abundance (and scarcity) on conflict. In the absence of well-enforced property rights, land markets are absent, thus removing the possibility of acquiring land with some income generated in another activity. As a consequence, claims to land are often made using violent means, at a cost (Skaperdas, 1992; Grossman and Kim, 1995; Garfinkel and Skaperdas, 2007), which may take the form of an opportunity cost of some foregone alternative income-generating activity. In such instances, our model predicts that a higher marginal valuation of small land-holdings is likely to imply increased violence when land becomes more scarce. This intuition is in line with empirical findings focusing on situations of extreme land scarcity. Homer-Dixon (1999) is perhaps the most well-known proponent of this neo-Malthusian thesis, though his supporting empirical material is deemed weak by the profession's standards. Yet, quantitative micro-econometric (Raleigh and Urdal, 2007; Brückner, 2010; Mwesigye and Matsumoto, 2016) and cross-country studies (Esteban et al., 2015) provide compelling evidence in support for the higher likelihood of conflict when the land-to-people ratio decreases. In the more specific case of the 1994 Rwandan genocide, anecdotal evidence suggests that land scarcity contributed to fuelling the intensity of the massacres (Newbury, 1998; Mamdani, 2001; Prunier, 2009). André and Platteau (1998) provide some tentative empirical results in support for this channel, while an in-depth econometric study of Verpoorten (2012) further confirms the phenomenon.

5.3. Foreign aid and rent seeking

Our model can be used to investigate the implications for rent seeking over foreign aid (e.g., Djankov et al., 2008). In many cases, foreign aid budgets are contested through the use of rent seeking. Foreign aid has often been considered a curse since it has been shown to result in a reduction of the provision of pubic goods, increases in corruption, and in weakening the recipient country's institutions

[10] This may not be the case in some production-appropriation contexts where only the opportunity cost of effort is considered, unless effort itself is distasteful (e.g., engaging in conflict).

[11] To be precise, consider a policy space on the real line and two lobbyists with preferred policy w^i and w^j respectively, where $w^i < w^j$. Each lobbyist decides on a level of lobbying effort x^i and x^j, and the position of the policy between w^i and w^j is determined by the proportional lobbying effort of the two parties, at position $w^i + \frac{x^j}{x^i + x^j}|w^j - w^i|$. As such, the contested rent may be seen as $w^j - w^i$, and how much of this rent is appropriated, by the location of the policy is given by $z^k = \frac{x^k}{x^i + x^j}[w^j - w^i]$, with $k = i, j$.

(Svensson, 2000; Knack, 2001), eventually resulting in damaging effects on economic growth (Economides et al., 2008). Investigations into the contestability of foreign aid and the implications for public good provisions and economic growth have previously been formulated using share contests (Svensson, 2000; Hodler, 2007). In such a contest, the share of foreign aid is distributed based on an agent's rent seeking relative to total outlays. Our approach can thus develop this literature by providing a more general theoretical setting by which to investigate the current research puzzles. In particular, we provide a link between the size of the foreign aid and the levels of rent seeking over its partition.

5.4. Campaign spending

A natural application of our framework is based on campaign spending: political parties invest in campaign spending in order to maximize voter share in an election. Indeed, share contests have previously been used to analyze the incentive to invest in campaign spending (Skaperdas and Grofman, 1995; Iaryczower and Mattozzi, 2012; Denter and Sisak, 2015). In this context, there is no *a priori* rationale for assuming political parties have quasi-linear utility in voter share. Indeed, one would expect that utility from voter share is non-linear and is, perhaps, quite complex. Depending on the specific voting rules applied, the (marginal) utility gained from certain voters may be very different from alternative voting systems, such as first-past-the-post and proportional representation. Our framework, by providing a general utility setting, can highlight new connections and interactions between campaign spending and voter share that can help assist in investigating the incentive to invest in campaign spending.

6. Conclusions

In microeconomic analysis, constant marginal utility and separable preferences are generally seen as a very special case. Yet, the study of share contests in the existing literature has assumed this as standard. In this article, we have extended the theory of contests to allow for general preferences and motivate our theory with an application to contests for public funds. With more general preferences the conventional wisdom of a monotonically increasing relationship between the contested rent (e.g., public funds) and effort (e.g., lobbying expenditures) expended in the contest—which proxies the social cost of rent-seeking—need no longer hold.

We take an aggregative games approach to study share contests with heterogeneous contestants that have general preferences. This allows us to deduce the uniqueness of Nash equilibrium in this more general framework and to undertake a tractable analysis of the properties of equilibrium. We show that the direction of change in the ratio of the marginal rate of substitution to contest allocation is crucial in determining whether effort increases or decreases when the contested rent increases. Our analysis allows us to understand the conditions on preferences under which the conventional wisdom does not hold and aggregate effort decreases when the contested rent increases.

Our framework opens up the applicability of contests to economic environments where the basic economic interaction corresponds to a share contest, but individuals' preferences are more sophisticated than the simple form that has so far been investigated within the contest literature. It is important to recognize that the conventional analysis that assumes linear utility functions is highly restrictive and inconsistent with many real-world situations. Thus, it would be misleading to draw conclusions about the relationship between the strategic variable (effort) and the contested economic rent based on the existing results in the contest literature. Our analysis offers a framework within which to understand the precise nature of this relationship.

Appendix A

Proof of Lemma 1. First, note that

$$MRS_z^i = -\frac{u_z^i u_{zx}^i - u_x^i u_{zz}^i}{\left[u_z^i\right]^2} \quad \text{and} \quad MRS_x^i = -\frac{u_z^i u_{xx}^i - u_x^i u_{zx}^i}{\left[u_z^i\right]^2}$$

and therefore $MRS_z^i \geq 0 \iff u_{zx}^i \leq -MRS^i u_{zz}^i$ and (noting that $u_x^i < 0$) $MRS_x^i \geq 0 \iff u_{zx}^i \leq -\frac{1}{MRS^i} u_{xx}^i$.

Quasi-concavity requires that the determinant of the first-order bordered Hessian is non-positive, and that the determinant of the second-order bordered Hessian is non-negative. The first condition is automatically satisfied as

$$\begin{vmatrix} 0 & u_z^i \\ u_z^i & u_{zz}^i \end{vmatrix} = -\left[u_z^i\right]^2 < 0.$$

The second condition requires

$$\begin{aligned} \begin{vmatrix} 0 & u_z^i & u_x^i \\ u_z^i & u_{zz}^i & u_{zx}^i \\ u_x^i & u_{zx}^i & u_{xx}^i \end{vmatrix} \geq 0 &\iff \\ -u_z^i \begin{vmatrix} u_z^i & u_{zx}^i \\ u_x^i & u_{xx}^i \end{vmatrix} + u_x^i \begin{vmatrix} u_z^i & u_{zz}^i \\ u_x^i & u_{zx}^i \end{vmatrix} \geq 0 &\iff \\ -u_z^i\left[u_z^i u_{xx}^i - u_x^i u_{zx}^i\right] + u_x^i\left[u_z^i u_{zx}^i - u_x^i u_{zz}^i\right] \geq 0 &\iff \\ \left[u_z^i\right]^2\left[u_z^i MRS_x^i - u_x^i MRS_z^i\right] \geq 0. & \end{aligned}$$

As such, since $MRS_z^i, MRS_x^i \geq 0$, $u(\cdot,\cdot)$ is quasi-concave.

Finally, as we note the restriction on second derivatives is equivalent to $u_{zz}^i \leq -\frac{1}{MRS^i} u_{zx}^i$ and $u_{xx}^i \leq -MRS^i u_{zx}^i$; thus, when $u_{zx}^i = 0$, $u_{zz}^i \leq 0$, $u_{xx}^i \leq 0$ and $u_{zz}^i u_{xx}^i - \left[u_{zx}^i\right]^2 \geq 0$, so $u^i(\cdot,\cdot)$ is concave. □

Proof of Lemma 2. Recall from Eq. (4) that a contestant's share function is given by $s^i(X;R) = \max\{0, \sigma^i\}$, where σ^i is the solution to

$$l^i(\sigma^i, X; R) \equiv MRS^i(\sigma^i R, \sigma^i X) - [1-\sigma^i]\frac{R}{X} = 0.$$

First, note that

$$l_\sigma^i = R\, MRS_z^i + X\, MRS_x^i + \frac{R}{X} > 0 \tag{7}$$

since $MRS_z^i, MRS_x^i \geq 0$ by Lemma 1, so for any given X and R there is a single value of σ^i where $l^i(\sigma^i, X; R) = 0$ so $s^i(X;R)$ is a function. Continuity is established from the assumed differentiability of the utility function. This establishes the first claim.

Envisaging $l^i(\sigma^i, X; R)$ plotted as an (increasing) function of σ^i, note that if $l^i(0, X; R) \geq 0$ then the fact that $l_\sigma^i > 0$ implies the value of σ^i where $l^i(\sigma^i, X; R) = 0$ will be non-positive, and hence $s^i(X;R) = 0$ (i.e., a corner solution); conversely, if $l^i(0,X;R) < 0$ then the value of σ^i where $l^i(\sigma^i, X; R) = 0$ will be positive so $s^i(X;R) > 0$ (i.e., an interior solution), and it will naturally not exceed one since $l^i(1, X; R) = MRS^i(R, X) > 0$.

Now, $l^i(0, X; R) = MRS^i(0,0) - R/X$. If $MRS^i(0,0) > 0$ then we can define the value of X where $l^i(0,X;R)$ is precisely zero by $\bar{X}^i \equiv R/MRS^i(0,0)$, which is strictly positive since $MRS^i(0,0) < \infty$ by assumption. As such, for any $X \geq \bar{X}^i$, $l^i(0,X;R) \geq 0$ and so by the reasoning in the previous paragraph $s^i(X;R) = 0$. Conversely, when

$0 < X < \bar{X}^i$, $l^i(0,X;R) < 0$ and so $s^i(X;R) > 0$. On the other hand, if $MRS^i(0,0) = 0$ then $l^i(0,X;R) = -R/X < 0$ for all $X < \infty$ and so $s^i(X;R) > 0$ for all $X < \infty$ (again by the reasoning in the previous paragraph) being defined by the solution in σ^i to $l^i(\sigma^i,X;R) = 0$. Now, as $X \to \infty$, $[1-\sigma^i][R/X] \to 0$ and so for $l^i(\sigma^i, \lim_{X\to\infty} X; R) = 0$ we need $\lim_{X\to\infty} MRS^i(\sigma^i R, \sigma^i X) = 0$. If $\lim_{X\to\infty}\sigma^i X > 0$ then part (b) of Assumption 1 implies $\lim_{X\to\infty} MRS^i(\sigma^i R, \sigma^i X) > 0$ so we must have $\lim_{X\to\infty}\sigma^i X = 0$, which requires $\lim_{X\to\infty}\sigma^i = 0$. As such, $\lim_{X\to\infty} s^i(X;R) = 0$. This demonstrates part (b) of our second claim.

Now, where it is positive (i.e., for $0 < X < \bar{X}^i$ if $MRS^i(0,0) > 0$ or for all $0 < X < \infty$ if $MRS^i(0,0) = 0$), the behavior of the share function as X changes, it being implicitly defined by $l^i(\sigma^i,X;R) = 0$, is given by

$$s_X^i = -\frac{l_X^i}{l_\sigma^i} = -\frac{\sigma^i MRS_x^i + [1-\sigma^i]\frac{R}{X^2}}{R\, MRS_z^i + X\, MRS_x^i + \frac{R}{X}} < 0 \quad (8)$$

again since $MRS_z^i, MRS_x^i \geq 0$ by Lemma 1, confirming the strict monotonicity in our third claim.

Finally we confirm the small X limit (part (a) of our second claim). Note that $Xl^i(\sigma^i,X;R) = XMRS^i(\sigma^i R, \sigma^i X) - [1-\sigma^i]R$ and since we assume $MRS^i(z^i,0) < \infty$ for all $z^i \geq 0$ (part (b) of Assumption 1) it follows that $\lim_{X\to 0} Xl^i(\sigma^i,X;R) = -[1-\sigma^i]R$. As such, $\lim_{X\to 0} l^i(0,X;R) < 0$ so we are seeking an interior solution, and the only possibility to achieve $l^i(\sigma^i,X;R) = 0$ as $X \to 0$ is $\sigma^i = 1$, implying $\lim_{X\to 0} s^i(X;R) = 1$. □

Proof of Lemma 3. We seek to show that X^* is a Nash equilibrium if and only if $S(X^*;R) = 1$. First, the 'if' part. If X^* is a Nash equilibrium then $x^{i^*} = b^i(X^{-i^*};R)$ for all $i \in N$. This implies $x^{i^*} = b^i(X^* - x^{i^*};R)$ which in turn implies $x^{i^*} = X^* s^i(X^*;R)$ for all $i \in N$, and therefore that $X^* = X^* \sum_{j\in N} s^j(X^*;R)$, and consequently $S(X^*;R) = 1$. For the 'only if' part, note that for each $i \in N$, $X^* s^i(X^*;R) = b^i(X^* - X^* s^i(X^*;R);R)$. If $S(X^*;R) = 1$ then $X^* = X^* S(X^*;R)$ and so for each $i \in N$, $X^* s^i(X^*;R) = b^i(X^* S(X^*;R) - X^* s^i(X^*;R);R) = b^i(X^{-i^*};R)$, thus allowing us to conclude that $x^{i^*} = X^* s^i(X^*;R)$ for all $i \in N$ constitutes a Nash equilibrium. □

Proof of Proposition 1. From Lemma 3, we know that Nash equilibria are identified by intersections of $S(X;R)$ with the unit line. From Lemma 2 we also know that individual share functions are single-valued, continuous and strictly decreasing in $X > 0$, and have the property $s^i(X;R) \to 1$ as $X \to 0$ and either $s^i(X;R) = 0$ for all $X \geq \bar{X}^i$ if $MRS^i(0,0) > 0$ or, if $MRS^i(0,0) = 0$, $s^i(X;R) \to 0$ as $X \to \infty$. As such, there exist two values of X, $\underline{X}$ and $\bar{X} > \underline{X}$, such that $S(\underline{X};R) > 1$ and $S(\bar{X};R) < 1$. Combined with the fact that $S(X;R)$ is continuous and strictly decreasing in $X > 0$, this implies there is a single value of X where $S(X;R) = 1$, and so a single Nash equilibrium exists. □

Proof of Lemma 4. Recall from Eq. (4) that a contestant's share function, where positive, is implicitly defined as the value of σ^i where

$$l^i(\sigma^i,X;R) \equiv MRS^i(\sigma^i R, \sigma^i X) - [1-\sigma^i]\frac{R}{X} = 0.$$

As such,

$$s_R^i = -\frac{l_R^i}{l_\sigma^i} = -\frac{\sigma^i MRS_z^i - [1-\sigma^i]\frac{1}{X}}{R\, MRS_z^i + X\, MRS_x^i + \frac{R}{X}}.$$

The denominator (as deduced in Eq. (7)) is positive. Noting that $\sigma^i R = z^i$ and that $[1-\sigma^i]\frac{R}{X} = MRS^i$ from the first-order condition, gives

$$s_R^i = -\frac{z^i MRS_z^i - MRS^i}{R[R\, MRS_z^i + X\, MRS_x^i + \frac{R}{X}]} = -w^i[\eta^i - 1]$$

where $w^i = MRS^i\left[R\left[R\, MRS_z^i + X\, MRS_x^i + \frac{R}{X}\right]\right]^{-1} > 0$, from where the statement in the lemma follows. □

Proof of Proposition 2. When $MRS^i(0,0) = 0$ for all $i \in N$, $s^i(X,R) > 0$ for all $X < \infty$ so in any equilibrium all players will be active. Implicit differentiation of Eq. (5) gives

$$\mathcal{X}'(R) = -\frac{\sum_{j\in N} s_R^j}{\sum_{j\in N} s_X^j}.$$

We deduced in Lemma 2 that $s_X^i < 0$ for all $i \in N$, and therefore $\text{sgn}\{\mathcal{X}'(R)\} = \text{sgn}\left\{\sum_{j\in N} s_R^j\right\}$. From Lemma 4 we know that, evaluated at the equilibrium, $s_R^i = -w^i[\eta^i(R) - 1]$, from where the statement in the proposition follows. □

Proof of Proposition 3. In a contest with rent R in which the set of active contestants is $\mathcal{N}(R)$, implicit differentiation of Eq. (5) assuming the set of contestants remains the same gives

$$\mathcal{X}'(R) = -\frac{\sum_{j\in\mathcal{N}(R)} s_R^j}{\sum_{j\in\mathcal{N}(R)} s_X^j},$$

and so $\text{sgn}\{\mathcal{X}'(R)\} = \text{sgn}\left\{\sum_{j\in\mathcal{N}(R)} s_R^j\right\}$, so if $\sum_{j\in\mathcal{N}(R)} w^j[\eta^j(R) - 1] \lesseqgtr 0$ and the set of contestants remains the same then $\mathcal{X}'(R) \gtreqless 0$.

In the case where aggregate effort of active participants increases, it may also be the case that previously inactive participants become active, and this supports the increase in aggregate effort. In the case where aggregate effort declines, our assumption rules out that previously inactive contestants could become active which might negate the result. The condition is sufficient but not necessary as changes that increase equilibrium aggregate effort could be driven by previously inactive participants becoming active. □

Appendix B. Supplementary online appendix

Supplementary online appendix to this article can be found online at https://doi.org/10.1016/j.jpubeco.2018.08.004.

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